

ASTRONOMISCHE NACHRICHTEN.

Nº 71.

Theoria nova Aberrationis Fixarum

auctore

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Formulae approximativae, qualibus dumtaxat astronomi ad calculandas aberrationes fixarum utuntur, consequenter tabulae quoque iisdem superstructae obnoxiae sunt erroribus, speciata, quam hodie passim affectare placet coelestium numerorum, praecisione haud spernendis, quoties stellae alterutri polo plani, cuius respectu aberrationes ipsarum investigantur, viciniores sunt. Formulae porro approximativae, tametsi in praxi semper sufficent, ex-

actarum neutiquam tollerent desiderium. Caremus denique in praesentem usque diem expressionibus analyticis aberrationum pro siderum positionibus ad^t horizontem relatis, cuiusmodi expressiones astronomia integratissima sua saltim gratia nullo non tempore postulabit. Tam illi desiderio, quam huic astronomiae postulato ubertim satisfacient sequentes geminae aequationes

$$\begin{aligned} \tan(a' - a) &= \frac{-\rho \cos B \sec b \sin(A-a)}{1 - \rho \cos B \sec b \cos(A-a)} = \frac{-\rho \cos B \sec b' \sin(A-a')}{1 + \rho \cos B \sec b' \cos(A-a')} \dots \dots \text{I} \\ \tan(b' - b) &= \left\{ \begin{array}{l} \frac{\rho \cos B \sin b' \cos \frac{1}{2}(2A-a'-a) \sec \frac{1}{2}(a'-a) - \rho \sin B \cos b}{1 - \rho \cos B \cos b' \cos \frac{1}{2}(2A-a'-a) \sec \frac{1}{2}(a'-a) - \rho \sin B \sin b} \\ \frac{\rho \cos B \sin b' \cos \frac{1}{2}(2A-a'-a) \sec \frac{1}{2}(a'-a) - \rho \sin B \cos b'}{1 + \rho \cos B \cos b' \cos \frac{1}{2}(2A-a'-a) \sec \frac{1}{2}(a'-a) + \rho \sin B \sin b'} \end{array} \right\} \dots \dots \text{II} \end{aligned}$$

in quibus characterum $a b a' b' A B \rho$ haec est positiva significatio:

$a b$ longitudo et borealis latitudo, aut ascensio recta et borealis declinatio, vel azimuthum a superiori meridiani medietate orientem versus computatum et altitudo supra horizontem, loci stellae, ab aberratione repurgati.

$a' b'$ correspondens longitudo et borealis latitudo, aut etc. loci eiusdem stellae aberratione contaminati.

$A B$ pariter correspondens longitudo et borealis latitudo, aut etc. directionis, directioni motus terrae anni circa Solem, aut diurni circa proprium axem oppositae.

ρ denique exponens rationis velocitatis alterutrius telluris motus ad velocitatem luminis.

Operae pretium est, aequationibus adductis I. II. confessim subjungere ipsarum demonstrationem, e genuinis quidem, quae acutissimo Gaussii ingenio debentur (Theor. Mot. Corpor. coelestium pag. 68) aberrationis fixarum notionibus petitam.

Utunque stupenda scilicet velocitate lumen propagetur, velocitas haec infinita magna haud est: aliud igitur erit temporis momentum t , dum radius lucis ab astro projectus centrum lentis objectivae telescopii, aliudque T , dum forum eiusdem lentis attingit: quatenus porro velocitates motuum terrae respectu velocitatis luminis prorsus contempnendae non sunt, alii quoque erunt in spatio concipiendi loci, f centri et g foci lentis objectivae tempore priori t , pariterque alii F et G tempore sequiori T : aliquid nullo negotio perspicitur, intra per exile tempusculum $T-t$ angulum FGf , qui aberratio lucis vocatur, aequalē esse angulo Gfg , qui evidenter est parallaxis puncti f , e punctis g et G simultanea visi: qua definitione aberrationes fixarum, aberrationis minimorum lucis in positiones siderum influxus, ad familiam algorithmumque parallaxium revocantur: ut sane mirandum veniat, Flamsteedio aliquique astronomis, qui aberrationes fixarum per annas ipsarum parallaxes explicare frusta adnisi sunt, ideam hanc, meditationibus suis adeo cognatam non illico occurrisse.

Pronus abhinc fit ad constructionem aequationum nostrarum gressus: ultiro enim patet, designari numeris

$$\begin{aligned} ab \text{ directionem rectae } Gf &= r, \text{ à } G \text{ versus } f \\ ab' &\quad gf = r', \text{ à } g \text{ versus } f \\ AB &\quad Gg = R, \text{ à } G \text{ versus } g. \end{aligned}$$

Habemus igitur, sive e duobus triangulis rectilineis, in quorum altero

$$\begin{aligned} \text{lateribus: } &r' \cos b', r \cos b, R \cos B \\ \text{anguli: } &a - A, 180^\circ - (a' - A), a' - a \end{aligned}$$

in altero autem

$$\begin{aligned} r' \cos b' \cos(a' + N) - r \cos b \cos(a + N) + R \cos B \cos(A + N) &= 0 \\ r' \cos b' \sin(a' + N) - r \cos b \sin(a + N) + R \cos B \sin(A + N) &= 0 \\ r' \sin b' &- r \sin b + R \sin B = 0 \end{aligned}$$

quae ternae aequationes, statuendo, ut hic nullo praecisionis detimento licet, $r = r'$, cum per definitionem ipsius sit $R = r\beta$, absunt tandem in has:

$$\begin{aligned} \cos b' \cos(a' + N) - \cos b \cos(a + N) + \beta \cos B \cos(A + N) &= 0 \\ \cos b' \sin(a' + N) - \cos b \sin(a + N) + \beta \cos B \sin(A + N) &= 0 \\ \sin b' &- \sin b + \beta \sin B = 0 \end{aligned}$$

Sponte iam in oculos incurrit usus foecunditasque harum aequationum. Ex obvia nimirum earundem combinatione manescimus formulas sequentes

$$\begin{aligned} \tan(a' + N) &= \frac{\cos b \sin(a + N) - \beta \cos B \sin(A + N)}{\cos b \cos(a + N) - \beta \cos B \cos(A + N)} \\ \tan(a' + N) &= \frac{\cos b' \sin(a' + N) + \beta \cos B \sin(A + N)}{\cos b' \cos(a' + N) + \beta \cos B \cos(A + N)} \\ \tan b' &= \frac{(\sin b - \beta \sin B) \sin(a' + N)}{\cos b \sin(a + N) - \beta \cos B \sin(A + N)} = \frac{(\sin b - \beta \sin B) \cos(a' + N)}{\cos b \cos(a + N) - \beta \cos B \cos(A + N)} \\ \tan b &= \frac{(\sin b' + \beta \sin B) \sin(a + N)}{\cos b' \sin(a' + N) + \beta \cos B \sin(A + N)} = \frac{(\sin b' + \beta \sin B) \cos(a + N)}{\cos b' \cos(a' + N) + \beta \cos B \cos(A + N)} \end{aligned}$$

adminiculo quarum exacta solutio problematis aberrationum fixarum, determinationis scilicet ipsarum $a' b'$ per $a b$ et vicissim, summa cum generalitate indicatur; valore arbitrario ipsius N efficiente, ut formularum postremaarum singula, mater sit innumerarum aliarum, e quarum deinde combinationibus variis denuo aliae exurgunt. Ne autem multitudine obrueremur, eas duntaxat in I et II stitimus, quas proposito scopo semper sufficere, simulque calculis numericis prae reliquis idoneas comperimus.

Ponendo scilicet in prima $N = -a$ et in secunda $N = -a'$, obtinuimus immediate aequationem I. E tertiae vero et quartae expressionibus anterioribus pro $N = -A$, aut posterioribus pro $N = 90^\circ - A$, consequuti sumus

$$\begin{aligned} \text{lateribus: } &r \sin b - R \sin B, r', r \cos b' \\ \text{anguli: } &b', 90^\circ, 90^\circ - b' \end{aligned}$$

ordine opponuntur; sive directius expeditiusque per applicationem theoriae coordinatarum orthogonalem, aequationes:

$$\begin{aligned} r' \cos b' \cos a' - r \cos b \cos a + R \cos B \cos A &= 0 \quad (1) \\ r' \cos b' \sin a' - r \cos b \sin a + R \cos B \sin A &= 0 \quad (2) \\ r' \sin b' &- r \sin b + R \sin B = 0 \end{aligned}$$

seu formando aggregata: (1) $\cos n \mp$ (2) $\sin n$ et (2) $\cos n \pm$ (1) $\sin n$, ubi n angulum valoris prorsus arbitrarii denotat, ac paucis reductionibus factis, ponendo $N = \mp n$, sequentes:

$$\begin{aligned} r' \cos b' \cos(a' + N) - r \cos b \cos(a + N) + R \cos B \cos(A + N) &= 0 \\ r' \cos b' \sin(a' + N) - r \cos b \sin(a + N) + R \cos B \sin(A + N) &= 0 \\ r' \sin b' &- r \sin b + R \sin B = 0 \end{aligned}$$

$$\begin{aligned} \cos b' \cos(a' + N) - \cos b \cos(a + N) + \beta \cos B \cos(A + N) &= 0 \\ \cos b' \sin(a' + N) - \cos b \sin(a + N) + \beta \cos B \sin(A + N) &= 0 \\ \sin b' &- \sin b + \beta \sin B = 0 \end{aligned}$$

$$\begin{aligned} \tan b' &= \frac{(\sin b - \beta \sin B) \sin(A - a)}{\cos b \sin(A - a)} \\ \text{et } \tan b &= \frac{(\sin b' + \beta \sin B) \sin(A - a)}{\cos b' \sin(A - a)} \end{aligned}$$

Quum porro calculus ipsarum $b' b$ per tangentes suas omnem, cuius tabulae nostrae trigonometriae capaces sunt, praecisionem deposceret, atque idcirco praetermodum opera evaderet, postremas duas formulas in illas transformavimus, quas II exhibet. Commodissime haec transformatio praestatur medio aequationis notissimae

$$\tan(b' - b) = \frac{\tan b' - \tan b}{1 + \tan b' \tan b}$$

Substitutis enim in hac aequatione (successive) valoribus ipsarum $\tan b'$ et $\tan b$, prodit post paucas easque facilioras reductiones expressio:

$$\text{tang } (b' - b) \left\{ \begin{array}{l} = \frac{2 \sin b \cos b \cos \frac{1}{2}(2A-a'-a) \sin \frac{1}{2}(a'-a) + \rho \sin B \cos b \sin(A-a')}{\sin(A-a) + 2 \cos^2 b \cos \frac{1}{2}(2A-a'-a) \sin \frac{1}{2}(a'-a) - \rho \sin B \sin b \sin(A-a')} \\ = \frac{2 \sin b' \cos b' \cos \frac{1}{2}(2A-a'-a) \sin \frac{1}{2}(a'-a) + \rho \sin B \cos b' \sin(A-a)}{\sin(A-a) - 2 \cos^2 b' \cos(A-a'-a) \sin \frac{1}{2}(a'-a) + \rho \sin B \sin b' \sin(A-a)} \end{array} \right\}$$

Hinc vero, eliminando $\sin(A-a')$, $\sin(A-a)$ medio aequationum

$$\begin{aligned} R \cos B \sin(a'-A) &= r \cos b \sin(a'-a) = 2r \cos b \sin \frac{1}{2}(a'-a) \cos \frac{1}{2}(a'-a) \\ R \cos B \sin(a-A) &= r' \cos b' \sin(a'-a) = 2r' \cos b' \sin \frac{1}{2}(a'-a) \cos \frac{1}{2}(a'-a) \end{aligned}$$

statuendoque, ut supra $r = r'$, formula nostra II.

Introducendo angulum auxiliarem u , adminiculo aequationis:

$$\text{tang}(b' - b) = \frac{-\rho \sin B \sec u \cos(b+u)}{1 - \rho \sin B \sec u \sin(b+u)} = \frac{-\rho \sin B \sec u \cos(b'+u)}{1 + \rho \sin B \sec u \sin(b'+u)} \dots \text{III.}$$

Proximum negotium iam nobis est in definiendis variis, quos in formulis nostris litterae A, B, ρ pro singulis modificationibus generalis problematis aberrationum obtinent. Sunto hunc in finem momento temporis, pro quo ab vel $a'b'$ assignantur atque aberrationes $a'-a$, $b'-b$ queruntur:

ϵ excentricitas orbitae solaris, in partibus semiaxis maioris eiusdem orbitae expressa.

w longitudo perigei solis.

Δ radius vector solis.

\odot longitudo Solis, seclusa aberratione.

ϵ obliquitas eclipticae.

M ascensio recta medii coeli.

ϕ latitudo loci terrestris, pro quo calculus aberrationis diurnae suscipitur.

$y = -206265'' e \sin(\odot - w)$.

$90^\circ + \odot + x + y$ et D ascensio recta et declinatio puncti eclipticae Q , cuius longitudo $= 90^\circ + \odot + y$.

μ, h azimuthum et altitudine ejusdem puncti.

$270^\circ + M + \eta$ et ψ longitudine et latitudine puncti aequatoris q , cuius ascensio recta $= 270^\circ + M$, id est, puncti cardinalis occidentis in loco, pro quo calculus aberrationis diurnae instituitur.

Atque perspicuum est, E, P, Z denotantibus polos eclipticae, aequatoris et horizontis, quantitates x, D, μ, h, η, ψ erui posse e triangulis sphaericis EPQ, ZPQ, EPq . Sunt nimurum anguli et latera, quorum usus eum in finem capietur, in characteribus supra introductis expressa

In triangulo primo

$EPQ = 180^\circ + \odot + x + y$, $PEQ = 360^\circ - (\odot + y)$, $EQ = 90^\circ$, $EP = \epsilon$, $PQ = 90^\circ - D$.

In triangulo secundo

$ZPQ = 90^\circ - (M - \odot - x - y)$, $PZQ = 180^\circ - \mu$, $ZQ = 90^\circ - h$, $EPQ = 90^\circ - D$, $ZP = 90^\circ - \phi$.

$\text{tang } u = \text{Cotg } B \cos \frac{1}{2}(2A-a'-a) \sec \frac{1}{2}(a'-a)$ definiendum, concinnior calculisque logarithmicis accommoda evadet formula II. Erit nimurum:

In triangulo tertio

$PEq = 360^\circ - (M + \eta)$, $EPq = M$, $Eq = 90^\circ - \psi$, $Pp = 90^\circ$, $EP = \epsilon$.

E formulis plurimis, quas triangula haec eodem scopo suppeditant, sequentes modo placuit annotare

$$\text{Cotg}(\odot + x + y) = \cos \epsilon \text{Cotg}(\odot + y)$$

$$\sin D = \sin \epsilon \cos(\odot + y)$$

$$\tan W = \tan D \operatorname{Cosec}(M - \odot - x - y)$$

$$\tan \mu = \cos W \text{Cotg}(M - \odot - x - y) \operatorname{Cosec}(\phi - W)$$

$$\tan h = \cos \mu \text{Cotg}(\phi - W)$$

$$\text{Cotg}(M + \eta) = \cos \epsilon \text{Cotg } M$$

$$\sin \psi = \sin \epsilon \cos M.$$

Caeterum ad obtainendos valores quantitatum x, D et ψ ne formulis quidem opus est, cum e tabulis reductionum punctorum eclipticae ad aequatorem, quales inter complures alias sunt XXI et XXIII Zachii in Tab. Solis Gothae 1792, medio argumentorum $90^\circ + \odot + y$ et $90^\circ + M$, directe deponi queant.

Fiet igitur, supponendo cum Delambre (Astronomie théorique et pratique Vol. III. p. 106), semiaxem maiorem orbitae solaris a lumine percurri intra $493'',2$ temp. solaris medii:

Pro aberratione annua

$$\rho = \frac{20,2543 \sqrt{1-e^2}}{206265 \Delta}$$

in longitudine et latitudine, $A = 90^\circ + \odot + y$,

$$B = 0$$

in ascensione recta et declinatione, $A = 90^\circ$

$$+ \odot + x + y, B = D$$

in azimutho et altitudine, $A = \mu$, $B = h$.

Pro aberratione diurna

$$\rho = \frac{0,31289 \cos \phi}{206265}$$

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in longitudine et latitudine, $A = 270^\circ + M + \eta$,
 $B = \psi$.

in ascensione recta et declinatione, $A = 270^\circ + M$,
 $B = 0$.

in azimutho et altitudine, $A = 270^\circ$, $B = 0$.

Neglecta porro excentricitate, quo facto fieri $\gamma = 0$ et
 $\Delta = 1$, neglectaque in I et II parte variabili denominatorum, in numeratoribus vero statuendo $a = a' = \alpha$,
 $b = b' = \delta$, ac tandem tangentes arcuum $a' - a$, $b' - b$
cum his ipsis permutando, orientur formulæ approximativæ
 $a' - a = -206265''\rho \cos B \sec d \sin(A - \alpha)$. . . I*
 $b' - b = +206265''\rho \{ \cos B \sin \delta \cos(A - \alpha) - \sin B \cos \delta \}$ II*
in quibus evidenter cunctæ, hactenus ab astronomis productæ, insuperque aliae nondum cognitæ velut in nuce comprehenduntur. Addi his potest, ut calculus ipsius
 $b' - b$ tabulis logarithmicis adaptetur, expressio:
 $b' - b = -206265''\rho \sin B \sec \omega \cos(\delta + \omega)$. . . III*

in quo angulus auxiliaris ω ex aequatione

$$\tan \omega = \cot B \cos(A - \alpha)$$

elicitur.

Formulae I p[re]l[imi] tunc solum proficia aut necessaria redditur applicatio, dum quantitas b vel b' ad 90° proxime accedit. Ab expressione vero II* ad II, aut a III* ad III recurrendum nec in hoc casu erit, neque in ullo alio. Formula igitur I pro casu illo peculiari et perquam raro, in quo scilicet e stellis conspicuis quoad aberrationem in ascensione recta et declinatione sola polaris hodie occurrit, in sua generalitate relicta; sufficiet hic formulas approximativas, ad varias problematis aberrationum modificationes applicatas, in quas scilicet I* et II* pro singulis litterarum $AB\rho$ valoribus abeunt, seorsim sistere; ut ex iis calculator astronomus eam, qua pro re nata opus habuerit, immediate depromere queat. Superfluum esset porro monere, in formulis hisce approximativis signo \odot designari longitudinem solis, ad aliquot minuta prima exactam.

(I) In longitudine α et latitudine δ

1) Aberratio annua

$$a' - a = -20'',2543 \sec \delta \cos(\odot - \alpha)$$

$$b' - b = -20'',2643 \sin \delta \sin(\odot - \alpha)$$

2) Aberratio diurna

$$a' - a = +0'',313 \cos \phi \sec \psi \sec \delta \cos(M + \eta - \alpha)$$

$$b' - b = +0'',313 \cos \phi \{ \cos \psi \sin \delta \sin(M + \eta - \alpha) - \sin \psi \cos \delta \}$$

ubi η et ψ e supra citatis tabulis *Zachii* medio argumenti
 $= 90^\circ + M$ absque interpolatione desumuntur.

(II) In ascensione recta α et declinatione δ .

1) Aberratio annua

$$a' - a = -20'',2543 \cos D \sec \delta \cos(\odot + x - \alpha)$$

$$b' - b = -20'',2543 \{ \cos D \sin \delta \sin(\odot + x - \alpha) + \sin D \cos \delta \}$$

ubi x et D ex iisdem tabulis *Zachianis* medio argumenti
 $= 90^\circ + \odot$ levi calamo depromuntur.

2) Aberratio diurna

$$a' - a = +0'',313 \cos \phi \sec \delta \cos(M - \alpha)$$

$$b' - b = +0'',313 \cos \phi \sin \delta \sin(M - \alpha)$$

(III) In azimutho α et altitudine δ

1) Aberratio annua

$$a' - a = -20'',2543 \cos h \sec \delta \sin(\mu - \alpha)$$

$$b' - b = +20'',2543 \{ \cos h \sin \delta \cos(\mu - \alpha) - \sin h \cos \delta \}$$

ubi, x et D , ut supra, inventis; quantitates μ et h eruuntur per aequationes:

$$\tan \omega = \tan D \cosec(M - \odot - x)$$

$$\tan \mu = \cos \omega \cotg(M - \odot - x) \cosec(\phi - \omega)$$

$$\tan h = \cos \mu \cotg(\phi - \omega)$$

2) Aberratio diurna

$$a' - a = +0'',313 \cos \phi \sec \delta \cos \alpha$$

$$b' - b = -0'',313 \cos \phi \sin \delta \sin \alpha$$

Facillime perspicitur, ob $\sin D = \sin s \cos \odot$, se-

cundam e formulis usitatissimis (II. 1) ponendo

$$c = -\frac{20'',2543}{2} \sin s \sin(\odot + \delta) - \frac{20'',2543}{2} \sin s \sin(\odot - \delta)$$

abiturem in sequentem

$b' - b = -20'',2543 \cos D \sin \delta \sin(\odot + x - \alpha) + c$
ut adeo pro $\alpha = 20'',2543 \cos D$ et $A = x$ formulae nostræ (II. 1) identicae sint cum *Gaussianis* in operis periodici: Monathliche Correspondenz v. *Zach* Vol. XVII.
pag. 316.

Perinde clarum est, statuendo

$$m = -20'',2543 \cos D, n = -20'',2543 \sin s \cos \odot$$

$$\text{et } \tan \omega = \frac{m}{n} \sin(\odot + x + \alpha)$$

formulas (II. 1)abituras in sequentes

$$a' - b = m \frac{\cos(\odot + x - \alpha)}{\cos \delta}$$

$$b' - b = n \frac{\cos(\delta - \omega)}{\cos \omega}$$

construique posse Tabulas aberrationis, calculis logarithmicis omnino accommodas et quae praeter longitudinem solis nullo alio argumento egeant. Prima talium tabularum columnæ præbebit $\log m$, qui identicus est cum $\log(-\alpha)$ in *Gaussianis* tabulis, secunda $\log n$, tertia numerum x , qui denuo identicus est cum numero A *Gaussiani*. Caeterum $\log n$ plerumque praestat calculo directo obtinere, quam e tabulis per interpolationes querere. Sed en Tabulas ipsas.

Tabula generalis Aberrationum Stellarum fixarum.

Argumentum Longitudo Solis = \odot

Gr.	Os			VII ^s			I ^s			VII ^s			II ^s			VIII ^s			Gr.
	$\log m$	$\log n$	x	$\log m$	$\log n$	x	$\log m$	$\log n$	x	$\log m$	$\log n$	x	$\log m$	$\log n$	x				
	—	— +	+	—	— +	+	—	— +	+	—	— +	+	—	— +	+				
0	1.2690	0.9066	0 0	1.2790	0.8442	2 11	1.2977	0.6056	2 6	1.3003	0.5326	1 53	1.3024	0.4407	1 32	30			
1	1.2690	0.9065	0 5	1.2796	0.8397	2 14	1.2982	0.5922	2 3	1.3012	0.4985	1 44	1.3032	0.3966	1 24	29			
2	1.2691	0.9063	0 11	1.2802	0.8351	2 16	1.2988	0.5782	2 0	1.3016	0.4802	1 40	1.3035	0.3726	1 20	28			
3	1.2691	0.9060	0 16	1.2808	0.8302	2 18	1.2993	0.5637	1 57	1.3020	0.4610	1 36	1.3039	0.3470	1 16	27			
4	1.2692	0.9056	0 22	1.2814	0.8252	2 20	1.2998	0.5485	1 54	1.3024	0.4407	1 32	1.3042	0.3196	1 11	26			
5	1.2692	0.9050	0 27	1.2821	0.8200	2 21	1.3003	0.5326	1 53	1.3047	0.4193	1 28	1.3050	0.2903	1 7	25			
6	1.2694	0.9042	0 32	1.2827	0.8146	2 23	1.3007	0.5160	1 47	1.3052	0.3966	1 24	1.3055	0.2587	1 3	24			
7	1.2696	0.9034	0 37	1.2833	0.8090	2 24	1.3012	0.4985	1 44	1.3055	0.3726	1 20	1.3057	0.2245	0 58	23			
8	1.2698	0.9024	0 43	1.2840	0.8032	2 25	1.3016	0.4802	1 40	1.3059	0.1872	0 53	1.3060	0.1872	0 53	22			
9	1.2700	0.9012	0 48	1.2846	0.7971	2 26	1.3020	0.4610	1 36	1.3061	0.9925	0 34	1.3065	0.9494	0 10	21			
10	1.2702	0.9000	0 53	1.2853	0.7909	2 27	1.3024	0.4407	1 32	1.3065	0.9494	0 10	1.3067	0.9066	0 0	20			
11	1.2705	0.8986	0 58	1.2859	0.7844	2 28	1.3028	0.4193	1 28	1.3070	0.1009	0 44	1.3072	0.0502	0 39	19			
12	1.2708	0.8970	1 3	1.2866	0.7777	2 28	1.3032	0.3966	1 24	1.3075	0.2245	0 58	1.3077	0.1872	0 53	18			
13	1.2711	0.8954	1 8	1.2872	0.7707	2 28	1.3035	0.3726	1 20	1.3080	0.1872	0 53	1.3082	0.1463	0 49	17			
14	1.2714	0.8935	1 12	1.2879	0.7636	2 28	1.3039	0.3470	1 16	1.3084	0.9066	0 25	1.3087	0.9066	0 0	16			
15	1.2717	0.8916	1 17	1.2886	0.7561	2 28	1.3042	0.3196	1 11	1.3090	0.9066	0 0	1.3092	0.9066	0 0	15			
16	1.2721	0.8895	1 22	1.2892	0.7484	2 28	1.3045	0.2903	1 7	1.3095	0.9066	0 0	1.3097	0.9066	0 0	14			
17	1.2725	0.8872	1 26	1.2899	0.7404	2 27	1.3047	0.2587	1 3	1.3100	0.9066	0 0	1.3102	0.9066	0 0	13			
18	1.2729	0.8848	1 30	1.2905	0.7321	2 27	1.3050	0.2245	0 58	1.3105	0.9066	0 0	1.3107	0.9066	0 0	12			
19	1.2733	0.8823	1 34	1.2911	0.7236	2 26	1.3052	0.1872	0 53	1.3110	0.9066	0 0	1.3112	0.9066	0 0	11			
20	1.2737	0.8796	1 39	1.2918	0.7147	2 25	1.3055	0.1463	0 49	1.3115	0.9066	0 0	1.3117	0.9066	0 0	10			
21	1.2742	0.8768	1 42	1.2924	0.7055	2 24	1.3057	0.1009	0 44	1.3120	0.9066	0 0	1.3122	0.9066	0 0	9			
22	1.2747	0.8738	1 46	1.2930	0.6960	2 22	1.3058	0.0502	0 39	1.3125	0.9066	0 0	1.3127	0.9066	0 0	8			
23	1.2752	0.8707	1 50	1.2936	0.6861	2 21	1.3060	9.9925	0 34	1.3130	0.9066	0 0	1.3132	0.9066	0 0	7			
24	1.2757	0.8774	1 53	1.2943	0.6758	2 19	1.3061	9.9258	0 30	1.3135	0.9066	0 0	1.3137	0.9066	0 0	6			
25	1.2762	0.8639	1 57	1.2949	0.6652	2 17	1.3062	9.8469	0 25	1.3140	0.9066	0 0	1.3142	0.9066	0 0	5			
26	1.2767	0.8603	2 0	1.2955	0.6542	2 15	1.3063	9.7502	0 20	1.3145	0.9066	0 0	1.3147	0.9066	0 0	4			
27	1.2773	0.8565	2 3	1.2960	0.6427	2 13	1.3064	9.6254	0 15	1.3150	0.9066	0 0	1.3152	0.9066	0 0	3			
28	1.2778	0.8526	2 6	1.2966	0.6308	2 11	1.3065	9.4494	0 10	1.3155	0.9066	0 0	1.3157	0.9066	0 0	2			
29	1.2784	0.8484	2 9	1.2972	0.6185	2 8	1.3065	9.1485	0 5	1.3160	0.9066	0 0	1.3162	0.9066	0 0	1			
30	1.2790	0.8442	2 11	1.2977	0.6056	2 6	1.3065	0.9066	0 0	1.3165	0.9066	0 0	1.3167	0.9066	0 0	0			
	—	+ —	—	—	+ —	—	—	+ —	—	—	+ —	—	—	+ —	—	—	Gr.		
	$\log m$	$\log n$	x	$\log m$	$\log n$	x	$\log m$	$\log n$	x	$\log m$	$\log n$	x	$\log m$	$\log n$	x				
	V ^s	XI ^s	IV ^s	V ^s	X ^s	III ^s	V ^s	XI ^s	IV ^s	V ^s	X ^s	III ^s	V ^s	XI ^s	IV ^s				

Usus tabulae.

Exemplum 1. In tabulis specialibus aberrationis et mutationis Franc. L. B. à Zach Gothae 1806 editis Vol. I. pag. XXIII habetur, pro $\alpha = 12^\circ 33' 56'',943$, $\delta = + 88^\circ 11' 10'',350$, qui numeri sunt ascensio recta et declinatio stellae polaris initio anni 1790 et pro longitudine solis $\odot = 10^\circ$, $a' - a = - 9' 48'',41$, $b' - b = + 0'',30$ Typus calculi quantitatum $a' - a$ et $b' - b$ per Tabulam nostram erit sequens: $x = + 0' 53''$

$\log m$	1.2702	n	0.9000	n
$\log \cos(\odot + x - \alpha)$	9.9998		$\log \cos(\delta - \omega)$. . . 8.5693
$C.D. \log \cos \delta$. . . 1.4996		$C.D. \log \cos \omega$. . . 0.0006
$\underline{\log (a' - a) . . . 2.7696 n}$			$\log (b' - b) =$	9.4699
$a' - a = -588''30 = -9^{\circ}48'',30$			$b' - b) =$	+ 0.295
$\log \frac{m}{n}$				
$\log \sin(\odot + x - \alpha)$	8.4678	n		
$\log \tan \omega$				
$\omega =$				$-3^{\circ}56'22''$

Exemplum 2. Calculus aberrationis in ascensione recta et declinatione α Cygni pro 17 Decemb. 1807. Pro epocha hac est $\alpha = 308^{\circ} 43' 15'' 75$, $\delta = + 44^{\circ} 35' 58'' 5$, $\odot = 8^{\text{h}} 25^{\text{m}} 9^{\text{s}}$, Tabula praebet $x = + 24'$, $\log m = 1.3063 n$, calculus vero directus $\log n = \log (- 20''.2543 \sin \epsilon) + \log \cos \odot = 9.8337$. En igitur Typum calculi

$\log 20''$	$,2543$
C. D. \log	206265
$\log \sqrt{(1 - ee)}$
C. D. $\log \Delta$
$\log p$
$\log \sin(A - a)$	9.99731
$\log \rho$	5.99088
$\log \cos B$	9.96670
$\log \sec b$	1.53226
$\log \cos(A - a)$	9.04501 n
$\cos \rho \cos B \sec b \cos(A - a)$	7.48715
C. D. $\log(1 - \rho \cos B \sec b \cos(A - a))$	9.99985
$\log tg(a' - a)$	7.48700 n
$a' - a$	- 10° 33' 05"

Nota. Solo numerorum intuita opus est, ut intellegatur, fine calculandarum aberrationum stellae polaris, nihil minorem exactitudinem praestituras esse formulas I* II* et III*, modo valori ex I* obtento addatur correctio

$$k = -\frac{206265'' \rho \cos^2 D \sec^2 b \sin(A-a) \cos(A-a)}{1 - \rho \cos D \sec b \cos(A-a)}$$

ut, siquidem $a' - a$ ex a' et b' calculata haberetur,

$\log m$	1.3063	n	9.8337
$\log \cos(\odot + x - \alpha)$	9.8629	$\log \cos(\delta - \omega)$	9.8672
C. D. $\log \cos \delta$	0.1475	C. D. $\log \cos \omega$	1.3082
$\log(a' - a)$	1.3167	n	1.0091
$a' - a$	- 20",74	$b' - b$	+ 10",21
$\log \frac{n}{m}$	1.4726	n	
$\log \sin(\odot + x - \alpha)$	9.8351	n	
$\log \operatorname{tg} \omega$	1.3077		
ω	= 87° 10'		

Tandem, ut usum quoque formularum I. II et III illustraremus, duo exempla numerica, per easdem calculata hic sistere placuit; primum quidem e coelo depromtum, alterum pro arbitrio effectum.

Exemplum 1. Calculus aberrationis annuae in ascensione recta et declinatione stellæ polaris 10. April. 1815, momento culminationis eiusdem, dum scilicet juxta *Besselum* ascensio recta media ipsius est $13^{\circ} 56' 39'',45$ et declinatio medio $= 88^{\circ} 19' 10'',6$. Longitudo solis momento huic respondet (in ephemeridibus berolinensibus) $= 19^{\circ} 41' 32'',$ unde denta aberratione solis $= - 20'',2$ prodit $\odot = 19^{\circ} 41' 52'',2$. Hinc $y = - 57' 2''$. Medio argumenti $= 90^{\circ}$ $+ \odot + y = 108^{\circ} 44' 50'',2$. Tabulae *Zachii* supra laudatae praebent $x = + 1^{\circ} 33' 25''$, ut scilicet sit $A = 111^{\circ} 18' 15''$ et $B = D = + 22^{\circ} 9' 6''$. Est porro $a = 13^{\circ} 56' 39'',45$ $+ \text{Nutat.} = 13^{\circ} 56' 8'',35$, et $b = + 88^{\circ} 19' 10'',6 + \text{Nutat.} = + 88^{\circ} 19' 3'',38$. Ac tandem $\log \Delta = 0.00115$. Typus igitur calculi juxta I et III erit sequens:

1.30652	$\log \operatorname{Ctg} B$	0.39029
4.68557	$\log \operatorname{Cos} \frac{1}{2}(2A-a'-a)$	9.05095 n
9.99994	$\log \operatorname{Sec} \frac{1}{2}(a'-a)$	0.00000
9.99885	$\log \operatorname{tang} u$	9.44124 n
5.99088	$u = -15^\circ 26' 27''$	
	$\log \operatorname{Cos}(b+u)$	9.46998
	$\log \rho$	5.99038
	$\log \operatorname{Sin} B$	9.57641
	$\log \operatorname{Sec} u$	0.01597
	$\log \operatorname{Sin}(b+u)$	9.98030
	$\log \rho \operatorname{Sin} B \operatorname{Sec} u \operatorname{Cos}(b+u)$	5.05324
	$C.D. \log(1 - \rho \operatorname{Sin} b \operatorname{Sec} u \operatorname{Sin}(b+u))$	0.00000
	$\log \operatorname{tg}(b'-b)$	5.05324 n
	$b'-b$	= 2'',33

$$k' = + \frac{206265'' \rho \rho \cos^2 D \sec^2 b' \sin(A-a') \cos(A-a)}{1 + \rho \cos D \sec b' \cos(A-a)}$$

quae correctiones facile in tabulas redigi possent, quarum argumentum sit longitudo solis. Atque, si hujusmodi tabulae praesto essent pro stella polari, Tabulae *Gaussianae* pro hac quoque refinari tuto possent per totum praesens saeculum. Compendia, quae calculus tabularum pro *ketk*,

aullo praecisionis detrimento admittit plura ultiro se offent. Ex. gr. in casu nostro est $k = -0'',19$: atque re ipsa neglecto denominatore in I* in obtinuissemus $a' - a = -10' 33'',24 = -10' 33'',05 - 0'',19$.

Exemplum 2. Quaeritur aberratio annua in longitudine et latitudine stellae, pro qua data sit longitudo media + Nutat. $325^\circ 33' 19'',22 = a$, latitudo media + Nutat. $= +89^\circ 59' 0'' = b$, die 1^o Januarii 1822 in meridie Berolinensi. Longitudo solis hoc momento in ephemeribus Berolinensibus notatur $= 280^\circ 33' 41'',$ quae

$\log 20'',2543$	1.3065168
$\log \sin 1''$	4.6855748
$\log \sqrt{1-ee}$	9.9999383
$C.D. \log \Delta$	0.0073584
$\log \rho$	5.9993883

$\log \sin(a-A)$	9.8494850 n
$\log \rho$	5.9993883
$\log \sec b$	3.5362739
$\log \cos(a-A)$	9.8494850
$\log \rho \sec b \sin(a-A)$	9.3851472 n
$C.D. \log(1-\rho \sec b \cos(a-A))$	0.1207569
$\log \tan(a'-a)$	9.5059041 n
$a'-a$	-17° 46' 24'',86
$\log \sin b$	0.0000000
$\log \xi$	5.9993883
$\log \cos \frac{1}{2}(A-a'-a)$	9.7703974
$\log \sec \frac{1}{2}(a'-a)$	0.0052450
$\log \cos b$	9.4637261
$\cos \rho \sin b \cos \frac{1}{2}(2A-a'-a) \sec \frac{1}{2}(a'-a)$	5.7750307
$C.D. \log(1-\rho \cos b \cos \frac{1}{2}(2A-a'-a) \sec \frac{1}{2}(a-a))$	0.0000000
$\log \tan(b'-b)$	5.7750307
$b'-b$	+ 12'',28728

Haud erit superfluum calculum problematis inversi subjugere; in quo scilicet caeteris exempli 2. conditionibus paribus, data sit longitudo apparenſ stellae $a' = 307^\circ 46' 54'',36$ et latitudo apparenſ $b = +89^\circ 59' 12'',28728$, juxta formulas, ex iisdem I. II pro $B = 0$ derivatas:

$\log \sin(a'-A)$	9.9490021 n
$\log \rho$	5.9993883
$\log \sec b'$	3.6357910
$\log \cos(a'-A)$	9.6603988
$\log \rho \sec b' \sin(a'-A)$	9.5841814 n
$C.D. \log(1+\rho \sec b' \cos(a'-A))$	9.9217227
$\log \tan(a'-a)$	9.5059041 n

exacte uti in problemate directo.

demſa aberratione solis $= -20'',6$ fit $280^\circ 34' 1'',6 = \odot$: hinc $y = -42'',38$, adeoque $A = 90^\circ + \odot + y = 10^\circ 33' 19'',22$ oblinetur. Habetur denique per easdem ephemerides $\log \Delta = 9.9926416$. Adhibebimus autem pro calculo formulas I et II, quae pro $B = 0$ abeunt in sequentes:

$$\tang(a'-a) = \frac{\rho \sec b \sin(a-A)}{1 - \rho \sec b \cos(a-A)}$$

$$\tang(b'-b) = \frac{\rho \sin b \cos \frac{1}{2}(2A-a-a) \sec \frac{1}{2}(a'-a)}{1 - \rho \cos b \cos \frac{1}{2}(2A-a-a) \sec \frac{1}{2}(a'-a)}$$

Typus igitur calculi erit, ut sequitur.

$\tang(a'-a) = \frac{\rho \sec b' \sin(a'-A)}{1 + \rho \sec b \cos(a'-A)}$	0.0000000
$\tang(b'-b) = \frac{\rho \sin b' \cos \frac{1}{2}(2A-a'-a) \sec \frac{1}{2}(a'-a)}{1 + \rho \cos b' \cos \frac{1}{2}(2A-a'-a) \sec \frac{1}{2}(a'-a)}$	0.0000000
$\log \sin b'$	0.0000000
$\log \rho$	5.9993883
$\log \cos \frac{1}{2}(2A-a'-a)$	9.7703974
$\log \sec \frac{1}{2}(a'-a)$	0.0052450
$\log \cos b'$	6.3642090
$\log \rho \sin b' \cos \frac{1}{2}(2A-a'-a) \sec \frac{1}{2}(a'-a)$	5.7750307
$C.D. \log(1 + \rho \cos b' \cos \frac{1}{2}(2A-a'-a) \sec \frac{1}{2}(a'-a))$	0.0000000
$\log \tan(b'-b)$	5.7750307
exacte ut in problemate directo.		

Sternbedeckungen.

In Bushey Heath (Breite $51^\circ 37' 44''$, Länge westlich in Zeit von Greenwich $1^\circ 20''9$) hat der Oberste Beaufoy folgende Eintritte beobachtet.

1824. Sept. 4. kleiner Stern Eintr. $19^h 10' 54,0$ Sternzeit.

Sept. 15. kleiner Stern Eintr. $3^h 28' 46,2$ —

Nov. 2. 19 Piscium Austr. $2^h 10' 13,8$ —

Jupiterstrabanten.

1824 Octbr. 2 Eintritt des 2^{ten} $13^h 50' 50''$ m. Z.

1824	Oct. 13	Eintr. d. 1 ^{ten}	$17^h 1' 8''$ m. Z.
—	17	Eintritt III	16 59 23 —
—	29	Eintritt I	15 16 4 —
Nov. 2		Eintritt II	13 26 35 —
—	5	Eintritt I	17 9 25 —
—	14	Eintritt I	13 30 48 —

Annals of Philos. Octbr.
Nov. Dec.